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Nonhamiltonian 2-connected claw-free graphs with large 4-degree sum

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Abstract

Let G be a 2-connected claw-free graph on n vertices. Let $\sigma_k(G)$ be the minimum degree sum among k-element independent set of vertices in G. It is proved that if $\sigma_4(G) \geqslant n+3$ then G is hamiltonian or else G belong to the known family of graphs. This is a generalization of the best known sufficient condition on hamiltonicity in claw-free 2-connected graphs given independently by Liu, Zhang and Broersma. Moreover, it is shown that the problem HAMILTONIAN CYCLE restricted to claw-free graphs $G=(V,E)$ with $\sigma_3(G) \geqslant \lfloor \frac{3}{4}(|G|+3) \rfloor$ has polynomial time complexity. This contrasts sharply with known results on NP-completeness among dense graphs.

1. Introduction

In this paper we consider only finite undirected graphs $G=(V(G),E(G))$ without loops and multiple edges. For terminology and notation not defined here we refer to [4]. For simplicity, we write $|G|$ instead of $|V(G)|$ for the order of G. We denote by $\langle H \rangle$ the subgraph of G induced by H if $H \subseteq V$. A graph G is said to be *traceable* [*hamiltonian*] if G has a hamiltonian path [hamiltonian cycle]. A graph G is *hamiltonian connected* if there exists a hamiltonian path between every pair of distinct vertices. A graph G is *homogeneously traceable* if every vertex is an endvertex of some hamiltonian path in G. A graph G is called *claw-free* if G includes no induced subgraph isomorphic to $K_{1,3}$. Many interesting properties of claw-free graphs are known (see [5]). The conditions on G we are going to deal with involve the parameter $\sigma_q(G)$ defined to be the *minimum sum of degrees* of q independent vertices taken over all independent q-subsets of V (due to a standard convention, $\sigma_q(G)=+\infty$ if no independent q-subset exists).

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Now, we define a family $\mathscr{F}$ of nonhamiltonian graphs which we denote

$$G_n = G_n(H_1, H_2, H_3). \tag{1}$$

Let G_n be a graph of order n ($\geqslant 9$) which can be decomposed into three disjoint subgraphs (of order $\geqslant 3$) H_1, H_2, H_3 such that for each $i = 1, 2, 3$ there exist exactly two different vertices $a_i, b_i \in H_i$, such that $E_G(H_i, H_j) = \{a_i a_j, b_i b_j\}$, where $i \neq j$, $1 \leqslant i, j \leqslant 3$. Let $\mathscr{F}'$ denote the subfamily of $\mathscr{F}$ these graphs $G_n(H_1, H_2, H_3)$ for which component graphs H_i, $i = 1, 2, 3$, are cliques.

The best-known sufficient condition for hamiltonicity using σ_3 for 2-connected claw-free graphs is the following.

Theorem 1 (Liu et al. [9], Zhang [14] and Broersma [3]). *If G is a 2-connected claw-free n-vertex graph such that*

$$\sigma_3(G) \geqslant n - 2, \tag{2}$$

then G is hamiltonian.

It is known that the condition (2) is best possible and it may be verified using graphs belonging to the family $\mathscr{F}'$.

Theorem 2 (Li [8]). *If G is a 2-connected claw-free n-vertex graph with $\delta(G) \geqslant n/4$, then either G is hamiltonian or $G \in \mathscr{F}$.*

Our aim is to prove a theorem which is a generalization of Theorem 2. Before we give a formulation of this theorem we have to introduce some notions.

For a vertex $x \in V(G)$, the set $N(x) := \{y \in V(G): xy \in E(G)\}$ is called the *neighbourhood* of x in G and $d(x) := |N(x)|$ is the degree of x in G. Additionally, let F and H be subgraphs of G. Then $N_F(x) := N(x) \cap V(F)$, $d_F(x) := |N_F(x)|$, $N(H) := \bigcup_{x \in V(H)} N(x) - V(H)$, $d(H) := |N(H)|$, $N_F(H) := \bigcup_{x \in V(H)} N_F(x)$ and $d_F(H) := |N_F(H)|$.

Now, we assume that a graph G is claw-free. If $\langle N(x) \rangle$ is a connected graph, we say that x is a *locally connected vertex*. A locally connected vertex with a noncomplete neighbourhood will be called an *eligible* vertex. For an eligible vertex $x \in V$, the operation of joining all pairs of nonadjacent vertices in $\langle N(x) \rangle$ by an edge will be called the *local completion* of G at x. In [11], Ryjáček proved that by a recurrent performing the local completion operation to eligible vertices of an arbitrary claw-free graph G until no such vertex remains, we get a claw-free graph which is uniquely determined by the graph G. This new graph is called the *closure* of G and is denoted by $\mathrm{cl}(G)$. By the construction of $\mathrm{cl}(G)$, the neighbourhood in $\mathrm{cl}(G)$ of every vertex is either a clique (if it is connected) or a disjoint union of two cliques (if it is disconnected). A claw-free graph G is called *closed* if $G = \mathrm{cl}(G)$.

Theorem 3 (Ryjáček [11]). *Let G be a claw-free graph. The graph G is hamiltonian if and only if $\mathrm{cl}(G)$ is hamiltonian.*

2. Main results

In Section 4 we prove the following result.

Theorem 4. *Let G be a* 2*-connected claw-free n-vertex graph. If*

$$\sigma_4(G) \geqslant n+3, \tag{3}$$

then either G is hamiltonian or $\mathrm{cl}(G) \in \mathscr{F}'$.

Corollary 5. *Let G be a* 2*-connected claw-free n-vertex graph satisfying* (3). *Then either G is hamiltonian or $G \in \mathscr{F}$. Moreover, if $G = G_n(H_1, H_2, H_3)$, then H_i is hamiltonian connected for each $i = 1, 2, 3$.*

Proof of Corollary 5. The first part of thesis is obvious because of Theorems 3 and 4. Now, it is easy to see that it is sufficient to consider the case when at least one of the H_i's (say H_1) is not a clique. Let x_1, y_1 be any different nonadjacent vertices from H_1. Then $n+3 \leqslant \sigma_4(G) \leqslant d(x_1)+d(y_1)+d(x_2)+d(x_3) \leqslant d_{H_1}(x_1)+d_{H_1}(y_1)+4+d(x_2)+d(x_3)$, where $x_i \in V(H_i) - \{a_i, b_i\}$, $i = 2, 3$. Hence, $d_{H_1}(x_1) + d_{H_1}(y_1) \geqslant |H_1| + 1$. Therefore, by the well-known theorem (cf. [4]), H_1 is hamiltonian connected. This ends the proof of the second part of thesis. □

As an immediate consequence we have the following statement.

Corollary 6. *Let G satisfy the assumptions of Corollary* 5. *Then G is homogeneously traceable.*

It is necessary to mention that perhaps the condition (3) in Theorem 4 is not the best possible (probably it may be replaced even by '$\sigma_4(G) \geqslant n$').

The above results have also some consequences in algorithmic theory of graphs. It is known that the classical problem HAMILTONIAN CYCLE restricted to claw-free graphs is NP-complete (see [2]). Moreover, Schiermeyer showed [12] that the problem HAMILTONIAN CYCLE remains NP-complete even if the problem is restricted to dense k-connected graphs which just fail to satisfy a sufficient condition for hamiltonicity, with σ_{k+1} is being involved. We show that dense claw-free graphs do not share this property with dense graphs in general case. It seems worth recalling that the author and Skupień in [7] showed that the similar situation holds for the problem HAMILTONIAN PATH in claw-free graphs. Let us consider the following problem.

HAMILTONIAN CYCLE (claw-free, $\sigma_3 \geqslant \lfloor \frac{3}{4}(n+3) \rfloor$)

Instance: a claw-free graph G of order n with $\sigma_3(G) \geqslant \lfloor \frac{3}{4}(n+3) \rfloor$.

Question: Is G hamiltonian?

Theorem 7. *The problem HAMILTONIAN CYCLE* (*claw-free*, $\sigma_3 \geqslant \lfloor \frac{3}{4}(n+3) \rfloor$) *has polynomial time complexity.*

Proof. Notice that if $\sigma_3(G) \geqslant \lfloor \frac{3}{4}(n+3) \rfloor$ then $\sigma_4(G) \geqslant n+3$. Therefore, by Theorems 3 and 4, G is nonhamiltonian if the closure of G is not 2-connected or belongs to the family $\mathscr{F}'$. It is obvious that we can find the closure of G and we can check whether a graph belongs to the family $\mathscr{F}'$ in polynomial time. Moreover, it is well-known that 2-connectedness of a graph can be checked in $O(n^2)$. This ends the proof. □

3. Preliminaries

Before we start the proof of the main theorem we mention some useful known theorems and lemmas.

Theorem 8 (Zhang [14] and Ainouche [1]). *Let G be a k-connected claw-free n-vertex graph. If $\sigma_{k+1}(G) \geqslant n-k$ then G is hamiltonian.*

Theorem 9 (Wu [13] and Flandrin and Li [6]). *Let G be a 3-connected claw-free n-vertex graph. If $\sigma_3(G) \geqslant n+1$ then G is hamiltonian connected.*

Let $\omega(G)$, $\kappa(G)$ and $\alpha(G)$ denote the number of components, the vertex connectivity and the independence number of a graph G, respectively. A graph G is said to be *tough* if $\omega(G-S) \leqslant |S|$ for every subset $S \subset V(G)$ with $\omega(G-S) > 1$. From the general Matthews and Sumner result proved in [8] we have the following.

Theorem 10. *If G is a 2-connected claw-free graph then G is tough.*

Lemma 11. *Let G be a claw-free graph. Let H be a subgraph of G and let $\{x_1, x_2, \ldots, x_k\}$ be an independent vertex set of G such that $\{x_1, x_2, \ldots, x_k\} \subseteq V(G) - V(H)$. Then*

$$\sum_{i=1}^{k} d_H(x_i) \leqslant 2|V(H)|.$$

Lemma 12. *Let G be a closed claw-free graph and let x be any vertex of G. Then*

(i) $\alpha(\langle N(x) \rangle) \leqslant 2$.
(ii) *if different vertices y and z from $N(x)$ are connected in $\langle N(x) \rangle$ by a path, then $yz \in E$.*

Lemma 13. *Let G be a closed claw-free graph. If $\sigma_2(G) \geqslant |G|+1$ then G is a clique.*

Proof. Suppose that there exist two nonadjacent vertices x and y in G such that $d(x)+d(y) \geqslant |G|+1$. Then $|N(x) \cap N(y)| \geqslant 3$ and, by claw-freeness of G, there exist two adjacent vertices u and v belonging to $N(x) \cap N(y)$. Therefore, x and y are connected by a path in $\langle N(u)\rangle$ (and $\langle N(v)\rangle$). Hence, we get a contradiction with Lemma 12(ii). $\square$

4. Proof of Theorem 4

Let G be a nonhamiltonian 2-connected claw-free graph of order n satisfying (3). Notice that the closure of G also satisfy the assumptions of Theorem 4 and, by Theorem 3, the closure of G is nonhamiltonian. Therefore, we may assume that G is closed. We are going to prove that $G \in \mathscr{F}'$.

First we notice that, by Theorem 8, the connectivity of G must be exactly 2. Let us denote by u_1 and u_2 cut-vertices of G and by G' and G'' the two (by Theorem 10) connected components of G after removing u_1 and u_2. Notice that at least one of the components G' and G'' is not a clique, because G is nonhamiltonian. On the other hand, if both G' and G'' are not cliques, then by Lemma 13, there exist nonadjacent vertices $x', y' \in V(G')$ and $x'', y'' \in V(G'')$ such that $d_{G'}(x') + d_{G'}(y') \leqslant |G'|$ and $d_{G''}(x'') + d_{G''}(y'') \leqslant |G''|$. Hence, by Lemma 11, $\sigma_4(G) \leqslant d(x') + d(y') + d(x'') + d(y'') \leqslant |G'| + |G''| + 2|\{u_1, u_2\}| \leqslant n + 2$, a contradiction with (3). Therefore, exactly one component, say G', is a clique. From now on, let V' be the set of vertices in G' and let x_0 denote an arbitrarily chosen vertex from V'.

Claim 14. *The connectivity of G'' is exactly* 2.

Proof. First, suppose that $\kappa(G'') \geqslant 3$. Then, by Theorem 9, there exists an independent triple of vertices $\{x_1, x_2, x_3\}$ in G'' such that $\sum_{i=1}^{3} d_{G''}(x_i) \leqslant |G''|$. Therefore, by Lemma 11, $\sigma_4(G) \leqslant \sum_{i=0}^{3} d(x_i) \leqslant |G'| - 1 + |G''| + 2|\{u_1, u_2\}| \leqslant n + 1$, which contradicts (3). Now, suppose that G'' is 1-connected. Let w denote a cut vertex of G'' and G_1'' and G_2'' be connected (by Theorem 10) components of $G'' - w$. Because G is 2-connected and claw-free, none of the vertices u_1, u_2 can have neighbours in both G_1'' and G_2''. Notice that if both G_1'' and G_2'' are cliques then G is hamiltonian. Therefore at least one component, say G_1'', is not a clique. By Lemma 13, there exist non-adjacent vertices $x_1'', y_1'' \in G_1''$, such that $d_{G_1''}(x_1'') + d_{G_1''}(y_1'') \leqslant |G_1''|$. Hence, by Lemma 11, $\sigma_4(G) \leqslant d(x_0) + d(x_1'') + d(y_1'') + d(x_2'') \leqslant |G'| - 1 + |G_1''| + |G_2''| - 1 + 2|\{u_1, u_2, w\}| \leqslant n + 1$, where x_2'' is any vertex from G_2''. This is a contradiction with (3). $\square$

Let us denote by v_1 and v_2 cut-vertices of G'' and by G_1 and G_2 the two (by Theorem 10) connected components of G'' after removing v_1 and v_2. Moreover, let V_1 and V_2 be the sets of vertices in G_1 and G_2, respectively. Now, we show that both G_1 and G_2 are cliques. Without loss of generality, suppose that G_1 is not a clique. Therefore, by Lemma 13, there exist nonadjacent vertices $x_1, y_1 \in V_1$, such that $d_{G_1}(x_1) + d_{G_1}(y_1) \leqslant |G_1|$. Hence, by Lemma 11, $d(x_0) + d(x_1) + d(y_1) + d(x_2) \leqslant |G'| - 1 + |G_1| + |G_2| - 1 + 2|\{u_1, u_2, v_1, v_2\}| \leqslant n + 2$, where x_2 is any vertex from G_2. This is a contradiction with (3).

Claim 15. (*by toughness and claw-freeness of G*)

(i) $N(u_i) \cap (V_1 \cup V_2) \neq \emptyset$, $i = 1,2$,
(ii) $u_i \notin N(V_1) \cap N(V_2)$, $i = 1,2$,
(iii) $|N(\{u_1,u_2\}) \cap (V_1 \cup V_2 \cup \{v_1,v_2\})| \geqslant 2$.

Claim 16. (*by toughness of G''*)
If $|V_i| \geqslant 2$ then $|N(\{v_1,v_2\}) \cap V_i| \geqslant 2$, where $i = 1,2$.

Claim 17. (*by Claim* 16)
For each $i \in \{1,2\}$ there exists a path from v_1 to v_2 containing all vertices from V_i.

Claim 18. (*by nonhamiltonicity of G*)
There is no path from u_1 to u_2 containing all vertices from $V_1 \cup V_2 \cup \{v_1,v_2\}$.

Without loss of generality, we can assume $N(u_1) \cap V_1 \neq \emptyset$. Certainly, by Claim 15, $N(u_1) \cap V_2 = \emptyset$. Let, for any $i,j \in \{1,2\}$,

$$v(i,j) := |(N(u_i) \cup N(v_j)) \cap V_i|.$$

Claim 19. *Let $i,j \in \{1,2\}$. If $v(i,j) > 1$ or $|V_i| = 1$ then there exists a path from u_i to v_j containing all vertices from V_i.*

We will consider two main cases.

Case 1: $N(u_2) \cap V_2 \neq \emptyset$.

It is obvious that $v(i,j) \geqslant 1$ for any $i,j \in \{1,2\}$.

Subcase 1.1: $v_1v_2 \in E$.

By Claim 19, if $v(1,1) > 1$ and $v(2,2) > 1$ (or, by symmetry, $v(1,2) > 1$ and $v(2,1) > 1$), then there exists a path $u_1 \ldots (V_1) \ldots v_1v_2 \ldots (V_2) \ldots u_2$, contrary to Claim 18.

Subcase 1.1.1: $v(1,1) = 1$ and $v(1,2) = 1$ (or, by symmetry, $v(2,2) = 1$ and $v(2,1) = 1$).

Then, by 2-connectedness of G'', $|V_1| = 1$. If $v(2,2) > 1$ or $v(2,1) > 1$ then, by Claim 19, we get a contradiction with Claim 18. Therefore $v(2,2) = 1$ and $v(2,1) = 1$, which implies, by 2-connectedness of G'', $|V_2| = 1$. Again, by Claim 19, we get a contradiction with Claim 18.

Subcase 1.1.2: $v(1,1) = 1$ and $v(2,1) = 1$ (or, by symmetry, $v(2,2) = 1$ and $v(1,2) = 1$).

Certainly $v(1,2) > 1$ and $v(2,2) > 1$. Hence $|V_1| > 1$. Therefore, by claw-freeness of G, $u_1v_1 \in E$. Using Claim 17 for V_1 and Claim 19 for V_2 we get a contradiction with Claim 18.

Subcase 1.2: $u_iv_j \in E$ for some $i,j \in \{1,2\}$.

By symmetry we may assume $u_1v_1 \in E$. Certainly, by claw-freeness, $u_1v_2 \notin E$. Notice that if $v(2,2) > 1$ or $|V_2| = 1$ then we get a contradiction similarly as in Subcase 1.1.2. Therefore $v(2,2) = 1$ and $|V_2| > 1$. Hence, by claw-freeness, $u_2v_2 \in E$,

$u_2v_1 \notin E$ and, by Claim 16, $v(2,1) > 1$. Let w_2 be the only one vertex from $(N(u_2) \cup N(v_2)) \cap V_2$. Similarly as before notice that $v(1,1) = 1$ and $|V_1| > 1$ because otherwise we get a contradiction using Claim 17 for V_2 and Claim 19 for V_1. Let w_1 be the only one vertex from $(N(u_1) \cup N(v_1)) \cap V_1$. Now, we show that G must be isomorphic to $G_n \in \mathscr{F}'$, where $a_1 = u_1$, $a_2 = v_1$, $a_3 = w_1$, and $b_1 = u_2$, $b_2 = w_2$, $b_3 = v_2$. It is enough to show that $\langle V' \cup \{u_1, u_2\}\rangle, \langle V_1 \cup \{w_1, v_2\}\rangle$ and $\langle V_2 \cup \{v_1, w_2\}\rangle$ are cliques. By symmetry, we prove it only for $\langle V' \cup \{u_1, u_2\}\rangle$. Let $w_i' \in V_i - \{w_i\}$, $i = 1, 2$. Suppose that there exist two nonadjacent vertices $x, y \in V' \cup \{u_1, u_2\}$. Hence, by Lemma 13, $\sigma_4(G) \leqslant d(x) + d(y) + d(w_1') + d(w_2') \leqslant |V'| + 2 + |\{v_1, v_2, w_1, w_2\}| + |V_1| + |V_2| = n + 2$, a contradiction.

Subcase 1.3: The opposite of the Subcase 1.2 holds.

By claw-freeness, $|V_1| > 1$, $|V_2| > 1$ and $v(i,j) > 1$ for each $i, j = 1, 2$.

Subcase 1.3.1: $d_{G_j}(v_i) = 1$ for $i, j = 1, 2$.

Let x_i be the neighbour of v_i in V_1 and let y_i be the neighbour of v_i in V_2, $i = 1, 2$. Then, by Lemma 11, $\sigma_4(G) \leqslant d(x_0) + d(x_1) + d(y_1) + d(v_2) \leqslant |V'| - 1 + |V_1| - 1 + |V_2| - 1 + 2|\{u_1, u_2, v_1\}| + 2 = n + 1$, a contradiction.

Subcase 1.3.2: For some $i, j \in \{1, 2\}$, $d_{G_j}(v_i) \geqslant 2$.

Without loss of generality, we may assume $d_{G_1}(v_1) \geqslant 2$. Because G_1 is a clique and $\langle V_1 \cup \{v_1\}\rangle$ is closed, $\langle V_1 \cup \{v_1\}\rangle$ is a clique. Since $v(1,2) > 1$, there exists a path from u_1 to v_2 containing all vertices from $V_1 \cup \{v_1\}$. Using Claim 19 for V_2 we get a contradiction with Claim 18.

Case 2: $N(\{u_1, u_2\}) \cap V_2 = \emptyset$.

Recall that, by Claim 15(i), $N(u_i) \cap (V_1 \cup V_2) \neq \emptyset$ for each $i = 1, 2$.

Subcase 2.1: $|N(\{u_1, u_2\}) \cap V_1| = 1$.

Let v be the only vertex from $N(\{u_1, u_2\}) \cap V_1$. By Claim 15(iii), $u_iv_j \in E$ for some $i, j \in \{1, 2\}$. Without loss of generality, we may assume $u_2v_2 \in E$. Now, by claw-freeness, $v_2v \in E$ and $N(v_2) \cap V_1 = \{v\}$. Therefore, by Claim 16, $v(1,1) > 1$ or $|V_1| = 1$. Hence, by Claim 19 for V_1 and Claim 17 for V_2, there exists a path $u_1 \ldots (V_1) \ldots v_1 \ldots (V_2) \ldots v_2u_2$, a contradiction with Claim 18.

Subcase 2.2: There exist three different vertices $x, y, z \in V_1$ such that $x \in N(u_1)$, $y \in N(u_2)$ and $z \in N(\{v_1, v_2\})$.

Without loss of generality, we assume that $z \in N(v_1)$. By Claim 16, there exists a vertex $t \in N(v_2) \cap V_1$ different from z. Without loss of generality, we may assume that $t \neq x$. Now it is easy to see that there exists a path $u_1x \ldots (V_1 - \{y, z, t\}) \ldots zv_1 \ldots (V_2) \ldots v_2tyu_2$, a contradiction with Claim 18.

Subcase 2.3: The opposite of the Subcase 2.2 holds.

Then $|N(\{u_1, u_2\})| = 2$ and $N(\{u_1, u_2\}) = N(\{v_1, v_2\})$. Let $N(\{u_1, u_2\}) = \{w_1, w_2\}$. Without loss of generality, we may assume that $w_1 \in N(u_1)$, $w_2 \in N(u_2)$, $w_1 \in N(v_1)$ and $w_2 \in N(v_2)$. Notice that if $|V_1| = 2$ then similarly as in Subcase 2.2 we get a contradiction with Claim 18. Hence $|V_1| > 2$. Therefore, by claw-freeness of G, $u_1v_1 \in E$ and $u_2v_2 \in E$. Notice that if $u_1v_2 \in E$ (or, by symmetry, $u_2v_1 \in E$) then there exists a path $u_1v_2 \ldots (V_2) \ldots v_1w_1 \ldots (V_1 - \{w_1, w_2\}) \ldots w_2v_2$, a contradiction with Claim 18. Hence u_1v_2, $u_2v_1 \notin E$ and, by claw-freeness, u_1w_2, u_2w_1, v_1w_2, $v_2w_1 \notin E$. Now, it is easy to check (similarly as in the end of the Case 1) that G must be isomorphic to $G_n \in \mathscr{F}'$, where $a_1 = u_1,\ a_2 = v_1,\ a_3 = w_1$, and $b_1 = u_2,\ b_2 = v_2,\ b_3 = w_2$. This ends the proof of Theorem 4. $\square$

5. Uncited reference

[10]

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